Permutations and Combinations

Subject: Mathematics –III
Subject Code EEE-405

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Objectives:

- apply fundamental counting principle
- permutations
- combinations

Fundamental Counting Principle

Fundamental Counting Principle can be used determine the number of possible outcomes when there are two or more characteristics .

Fundamental Counting Principle states that if an event has m possible outcomes and another independent event has n possible outcomes, then there are $m * n$ possible outcomes for the two events together.

Fundamental Counting Principle

Lets start with a simple example.

Suppose you have 3 pairs of shoes and 4 pair of socks. In how many ways can you have wear them ?

We have 3 pairs of shoes and 4 pairs of socks.
So, we can wear it 3×4

Fundamental Counting Principle

Mohan has 3 pants and 2 shirt. How many different pairs of a pant and a shirt, can he dress up with? There are 3 ways in which a pant can be chosen, because there are 3 pants available. Similarly, a shirt can be chosen in 2 ways. For every choice of a pant, there are 2 choice of a shirt, there there are

3*2 = 6 pair of a pant and a shirt

Permutations

A **Permutation** is an arrangement in a definite order of a number of object taken some or all at a time.

Notice, ORDER MATTERS!

To find the number of Permutations of n items, we can use the Fundamental Counting Principle or factorial notation.

Permutations

The number of ways to arrange

If we have to determine the number of 3 letters words, with or without meaning , which can be formed out of the letters of the word NUMBER, where the repetition of the letters is not allowed, we need to count the NUM, NMU, MUN, NUB,…..etc, Here we are counting the permutation of 6 different letters taken 3 at a time. The required number of words =$6\times 5 \times 4 = 120$.

Permutations

To find the number of Permutations of n items chosen r at a time, you can use the formula

$${}_{n}p_{r} = \frac{n!}{(n-r)!} \quad \text{where } 0 \le r \le n.$$

$${}_{5}p_{3} = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5*4*3 = 60$$

Permutations

Practice:

How many 4- digit numbers can be formed by using the digit 1 to 9 if repetition of digit is not allowed?

$$ {}_{9}p_{4} = \frac{9!}{(9-4)!} = \frac{9!}{5!} = 9 \times 8 \times 7 \times 6 = 3024 $$

Permutations

Practice:

From a club of 24 members, a President, Vice President, Secretary, Treasurer and Historian are to be elected. In how many ways can the offices be filled?

$$_{24}p_5 = \frac{\mathbf{24!}}{\mathbf{(24-5)!}} = \frac{24!}{19!} =$$

$$\mathbf{24 * 23 * 22 * 21 * 20 = 5,100,480}$$

Combinations

A **Combination** is an arrangement of items in which order does not matter.

ORDER DOES NOT MATTER!

Since the order does not matter in combinations, there are fewer combinations than permutations. The combinations are a "subset" of the permutations.

Combinations

To find the number of Combinations of n items chosen r at a time, you can use the formula

$$_{n}C_{r} = \frac{n!}{r!(n-r)!} \quad \text{where } 0 \le r \le n.$$

Combinations

To find the number of Combinations of n items chosen r at a time, you can use the formula

$$_nC_r = \frac{n!}{r!(n-r)!} \quad \text{where } 0 \le r \le n.$$

$$_5C_3 = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!} =$$

$$\frac{5*4*3*2*1}{3*2*1*2*1} = \frac{5*4}{2*1} = \frac{20}{2} = 10$$

Combinations

Practice: To play a particular card game, each player is dealt five cards from a standard deck of 52 cards. How many different hands are possible?

$$_{52}C_5 = \frac{52!}{5!(52-5)!} = \frac{52!}{5!47!} =$$

$$\frac{52*51*50*49*48}{5*4*3*2*1} = 2,598,960$$

Combinations

Practice: A student must answer 3 out of 5 essay questions on a test. In how many different ways can the student select the questions?

$$ {}_{5}C_{3} = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!} = \frac{5*4}{2*1} = 10 $$

Combinations

Practice: A basketball team consists of two centers, five forwards, and four guards. In how many ways can the coach select a starting line up of one center, two forwards, and two guards?

Center:

$$ {}_2C_1 = \frac{2!}{1!1!} = 2 $$

Forwards:

$$ {}_5C_2 = \frac{5!}{2!3!} = \frac{5*4}{2*1} = 10 $$

Guards:

$$ {}_4C_2 = \frac{4!}{2!2!} = \frac{4*3}{2*1} = 6 $$

$$ {}_2C_1 * {}_5C_2 * {}_4C_2 $$

Thus, the number of ways to select the starting line up is 2*10*6 = 120.

THANKS